LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2023**

UMT 4501 – REAL ANALYSIS-I

Date: 02-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	**Answer the following**
a)	Recall a finite set.
b)	Determine the set of all real numbers x such that $2x + 3 \leq 6$.
c)	Describe Archimedean property.
d)	Define limit of a sequence.
e)	Write comparison test for series.
2.	**Fill in the blanks**
a)	The function $f: A \rightarrow B$ is said to be injective, if ___________.
b)	If $x \geq -1$, then $(1 + x)^n \geq$___________ for all $n \in N$.
c)	If $S_3 = \left\{\frac{1}{n}: n \in N\right\}$ then $sup\ S_3 =$ ______.
d)	A sequence (x_n) such that $x_n \geq x_{n+1}, n \in N$ is called _____________.
e)	If $\sum x_n$ is convergent then $lim(x_n) =$ ___________.
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**Choose the correct answer**
a)	If there exists a bijection between N and S then S is known as ___________ set. (i) uncountable (ii) finite (iii) infinite (iv) denumerable
b)	The sum of two rational numbers is __________. (i) rational (ii) irrational (iii) integer (iv) natural number
c)	A function is said to be bounded if __________ for some $M \in R$. (a) $\|f(x)\| \geq M$ (ii) $\|f(x)\| = M$ (iii) $\|f(x)\| > M$ (iv) $\|f(x)\| \leq M$
d)	$lim\left(\frac{2n}{n^2+1}\right) =$ __________. (i) 0 (ii) 1 (iii) 2 (iv) 3
e)	The series $\sum \frac{1}{n^p}$ converges if ___________. (i) $p = 1$ (ii) $p > 1$ (iii) $p < 1$ (iv) $-1 < p < 1$
4.	**True or False**
a)	If S is a finite set then the number of elements in S is not a unique number in N.
b)	If z and a are elements in R with $z + a = a$ then $z = 0$.
c)	The infimum of the set $A = \{x \in R: 2x + 5 > 0\}$ is $\frac{5}{2}$.
d)	A sequence of real numbers is said to be monotone if it neither increasing nor decreasing.

e)	Let $X = (x_n)$ be a sequence in R then $\sum x_n$ is absolutely convergent if $\sum \lvert x_n \rvert$ is convergent in R.
SECTION B - K3 (CO2)	
	Answer any TWO of the following (2 x 10 = 20)
5.	Prove that the set of all rational numbers is denumerable.
6.	For all $a, b \in R$, show that (i) $\lvert ab \rvert = \lvert a \rvert \lvert b \rvert$ (ii) $\lvert a \rvert^2 = a^2$ (iii) if $c \geq 0$ then $\lvert a \rvert \leq c$ if and only if $-c \leq a \leq c$.
7.	Is [0,1] uncountable? Justify it.
8.	Examine whether the sequence $\left(1 + \frac{1}{n}\right)^n$ converges to Euler number $2 < e < 3$.
SECTION C – K4 (CO3)	
	Answer any TWO of the following (2 x 10 = 20)
9.	State and prove Cantor's theorem.
10.	Conclude that there is no rational number r such that $r^2 = 2$ with a constructed proof.
11.	"Prove that a sequence in R can have at most one limit"
12.	Test the convergence for the series $\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdot \ldots \cdot 2n}{1 \cdot 3 \cdot 5 \ldots \cdot (2n+1)}$ using Raabe's test.
SECTION D – K5 (CO4)	
	Answer any ONE of the following (1 x 20 = 20)
13.	(a) Consider the sets S and T such that $T \subseteq S$, then justify the following statements. (i) If S is a finite set then T is a finite set. (ii)If T is an infinite set then S is an infinite set. (10 marks) (b) Express the relation between Arithmetic mean and Geometric mean in the form of inequality and defend it with the proof. (10 marks
14.	(a) Defend Nested interval property with a suitable proof. (10 marks) (b) Let $Z = \{z_n\}$ be a decreasing sequence of strictly positive numbers with $lim(z_n) = 0$. Then prove that the alternating series $\sum_{n=0}^{\infty} (-1)^{n+1} z_n$ is convergent. (10 marks)
SECTION E – K6 (CO5)	
	Answer any ONE of the following (1 x 20 = 20)
15.	(a) Let $X = \{x_n\}$ and $Y = \{y_n\}$ be sequences of real numbers that converges to x and y respectively and let $c \in R$ then construct a proof such that the sequences $X + Y$, XY and cX converges to $x + y$, xy and cx respectively. (12 marks) (b) Let $X = \{x_n\}$ be sequences of real numbers that converges to x and $Z = \{z_n\}$ is a sequence of non-zero real numbers that converges to z and if $z \neq 0$ then show that the quotient sequence X/Z converges to x/z. (8 marks)
16.	(a) Sketch the proof for the statement that "a number u is the supremum of a nonempty subset S of $\mathbb{R}$ if and only if u satisfies the conditions (i) $s \leq u$ for all $s \in S$ and (ii) if $v < u$ then there exists $s' \in S$ such that $v < s'$". (10 marks) (b) Test the convergence of the series $\sum_{n=1}^{\infty} a_n cosnx$ by Dirichlet's test. (10 marks)

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